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Then $P=\frac{S}{R-1}\left(\frac{R^n-1}{R^n}\right)$, whence $S=\frac{rPR^n}{R^n-1}$, where $R=1+r$. If we assume $r=.0$, we find $S=\$149.0294$ nearly.

The limit of B's life is $4385\div75=58\frac{7}{15}$ years, $=l$. $\therefore$ the probability that B will be living at the end of 1 year is $\frac{l-1}{l}$; at the end of two years, $\frac{l-2}{l}$; at the end of three years, $\frac{l-3}{l}$; etc.

The present worth of $S,=\$149.20207$, due in 1, 2, 3, 4, etc., years is $\frac{S}{R}$, $\frac{S}{R^2}$, $\frac{S}{R^3}$, etc., to $\frac{S}{R^n}$. The present worth of S due at the end of any year multiplied by the probability of B's living to the end of that year is the actual value of S.

$$\therefore\ S'=\frac{S(l-1)}{Rl}+\frac{S(l-2)}{lR^2}+\frac{S(l-3)}{lR^3}+\ldots\ldots\frac{S(l-n)}{lR^n}\ldots\ldots(1).$$

$$\frac{S'}{R}=\frac{S(l-1)}{lR^2}+\frac{S(l-2)}{lR^3}+\frac{S(l-3)}{lR^4}\ldots\ldots\frac{S(l-n)}{lR^{n+1}}\ldots\ldots(2),\text{ by multiplying (1) by }\frac{1}{R}.$$

$$\therefore\ S'\left(\frac{R-1}{R}\right)=\frac{S(l-1)}{Rl}-\frac{S}{lR^2}-\frac{S}{lR^3}-\ldots\ldots-\frac{S(l-n)}{lR^{n+1}},$$

$$=\frac{S}{l}\left\{\frac{l}{R}-\frac{1}{R}-\frac{1}{R^2}-\frac{1}{R^3}\ldots\ldots\frac{1}{R^n}-\frac{l-n}{R^{n+1}}\right\},$$

$$=\frac{S}{l}\left\{\frac{l}{R}-\frac{l-n}{R^{n+1}}-\frac{1}{R}-\frac{1}{R^2}\ldots\ldots\frac{1}{R^n}\right\},$$

$$=\frac{S}{l}\left\{\frac{l}{R}-\frac{l-n}{R^{n+1}}-\frac{1}{R}\left[1-\frac{1}{R^n}\right]\div\frac{R-1}{R}\right\},$$

$$=\frac{S}{R}\left\{\left(1-\frac{l-n}{lR^n}\right)-R\left(1-\frac{1}{R^n}\right)\div(R-1)l\right\},$$ whence $S'=S/r\{[1-(l-n)/lR^n]-R(1-1/R^n)/rl\},=\$911.881029$.

No solutions of this problem were received from our contributors.

18. Proposed by H. W. DRAUGHON, Clinton, Louisiana.

The probability that A will speak the truth is twice the probability that B will, in an independent statement, speak the truth; but, if A exerts his influence, the probability is that B will agree with him in any statement. What is the probability of the truth of their concurrent testimony, the chances being equal that A may or may not be interested in the matter?

Solution by P. H. PHILBRICK, C. E., Lake Charles, Louisiana.

1. Suppose that A is not interested in the matter. Let x=the probability of the truth of any one of B's statements. Then $2x$=the probability of the truth of any one of A's statements. The event did occur if both witnesses tell the truth the probability of which is $x\times2x=2x^2$.

The event did not occur if both witnesses testify falsely the probability of which is $(1-x)(1-2x)$. Hence the probability of the occurrence of the event supposing x to be known is, $p'=\dfrac{2x^2}{2x^2+(1-x)(1-2x)}$.

Now as the veracity of A may vary from 0 to 1 x, may vary from 0 to $\frac{1}{2}$, and, therefore, the required probability is

$$p=\int_0^{\frac{1}{2}}p'dx\div\int_0^{\frac{1}{2}}dx=4\int_0^{\frac{1}{2}}\frac{x^2dx}{2x^2+(1-x)(1-2x)}=64\int_0^{\frac{1}{2}}\frac{x^2dx}{(8x-3)^2+7}.$$

Let $8x-3=y$, then $x=\frac{1}{8}(y+3)$, $dx=\frac{1}{8}dy$ and the limits of y are 1 and -3.

$$\text{Hence, } p=\tfrac{1}{8}\int_{-3}^{+1}\frac{(y+3)^2dy}{y^2+7}=\tfrac{1}{8}\int_{-3}^{+1}\left(1+\frac{6y}{y^2+7}+\frac{2}{y^2+7}\right)dy$$

$$=\left[\tfrac{1}{8}y+\tfrac{3}{8}\log_e(y^2+7)+\frac{1}{4\sqrt{7}}\tan^{-1}\frac{y}{7}\right]_{-3}^{1}=\tfrac{1}{2}-\tfrac{3}{8}\log_e2+\frac{1}{4\sqrt{7}}\tan^{-1}\sqrt{7}.$$

2. Suppose that A is interested in the matter. In this case B's testimony agrees with that of A, and A's alone is to be considered. The probability of the truth of A's testimoney is $P'=2x$ in which x may vary from 0 to $\frac{1}{2}$. Hence, the required probability is $P=\int_0^{\frac{1}{2}}P'dx\div\int_0^{\frac{1}{2}}dx=\left[2x^2\right]_0^{\frac{1}{2}}=\frac{1}{2}$.

Since the chances are equal that A may be or may not be interested in the matter, the probability required is equal to the half sum of the preceeding results, or $p_1=\frac{1}{2}\left(1-\frac{3}{8}\log_e2+\dfrac{1}{4\sqrt{7}}\tan^{-1}\sqrt{7}\right)$.

PROBLEMS.

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28. Propssed by G. I. HOPKINS, Instructor in Mathematics and Physics in High School, Manchester, N. H.

John and Henry passed in to their teacher a written exercise in geometry. On glancing over the two papers, the teacher noticed a striking resemblance in the hand writing. On closer inspection, he found the two diagrams precisely alike each employing the same nine letters, the points being designated by the same letters in both. The teacher at once suspected that Henry had asked and received assistance from John, but refraines from saying anything until he had examined the papers thoroughly. On further inspection of John's paper he discovered two mistakes: then turning to Henry's paper he discovered the same two mistakes in his. The next morning John denied that he had written that exercise for Henry but acknowledged that he had done so once before. When asked by the teacher, however, to pick out his paper from a number of others, the autographs all being concealed, he picked out the paper to which Henry's name was signed. Henry was absent from school that morning, and has not yet returned. What is the probability that John did *not* write Henry's exercise for him?